Singularities in and Stability of Ooguri-Vafa-Verlinde Cosmologies

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ABSTRACT

Ooguri, Vafa, and Verlinde have recently proposed an approach to string cosmology which is based on the idea that cosmological string moduli should be selected by a Hartle-Hawking wave function. They are led to consider a certain Euclidean space which has *two different Lorentzian interpretations*, one of which is a model of an *accelerating cosmology*. We describe in detail how to implement this idea without resorting to a "complex metric". We show that the four-dimensional version of the OVV cosmology is null geodesically incomplete but has no curvature singularity; also that it is [barely] stable against the Seiberg-Witten process [nucleation of brane pairs]. The introduction of matter satisfying the Null Energy Condition has the paradoxical effect of both stabilizing the spacetime and rendering it genuinely singular. We show however that it is possible to arrange for an *effective* violation of the NEC in such a way that the singularity is avoided and yet the spacetime remains stable. The possible implications for the early history of these cosmologies are discussed.

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1. The Perils of Ooguri-Vafa-Verlinde Cosmologies

Ooguri, Vafa, and Verlinde have put forward [1] an approach to string cosmology which implements the "wave function of the Universe" programme of Hartle and Hawking [2]. They do not propose a specific four-dimensional cosmological model, but here we shall see that much can be said about "OVV cosmology" [see also [3]] even at this early stage.

Ooguri et al begin by emphasising that it is natural, in the context of quantum gravity, to assume that the spatial sections of our universe are compact, a point also recently stressed by Linde [4][5] from a different point of view. This assumption plays a crucial role in the Ooguri et al proposal, since it means that various string moduli are not fixed — they must be selected by a wave function with amplitudes peaked at the appropriate moduli values.

Ooguri et al work with a compactification to Euclidean two-dimensional anti-de Sitter space, the hyperbolic space H^2; they use the foliation of H^n, familiar from studies of the AdS/CFT correspondence, by *flat* slices [6]. In the case of H^2, they use this foliation to perform a further compactification of H^2 to a space with topology $I\!R \times S^1$, where S^1 is a circle — or, rather, a one-dimensional torus. The discussion in [1] is based entirely on this two-dimensional space and the *two different* Lorentzian spacetimes which Ooguri et al derive from it.

Ultimately, of course, Ooguri et al hope to obtain a quasi-realistic *four-dimensional* cosmology in this way. We argue that, whatever the precise form of this spacetime ultimately proves to be, one should expect the topology of its spatial sections to be that of a *torus* [or a quotient of a torus]. There are several reasons for this. First, it would allow us to make contact with the work of Linde [4][5] mentioned above; second, it is compatible with the well-known Brandenberger-Vafa string gas cosmologies [7]; third, granted compactness, the observed [8] near-flatness of our Universe also points to toral spatial sections. Finally, toral spatial sections, if they are of the right size, allow us to answer many of the objections which have been raised against the existence of horizons in accelerating cosmologies. Note that the size of the tori is not determined by the spacetime curvature, so it must be fixed in some other way[2]; one can hope that it is determined by the wave function of the Universe. The fundamental importance of this spatial size "modulus" will appear at several points in our work. [See [9] for an extended discussion of various aspects of spacetimes with flat, compact spatial sections.]

The simplest way to proceed towards four dimensions is as follows. The standard foliation of H^4 by flat sections can be used to define a partial compactification of H^4 to a space with topology $I\!R \times T^3$, where T^3 is the three-torus. This is exactly analogous to the OVV procedure, and so we shall assume that four-dimensional space and spacetime have this structure. This simple assumption has far-reaching consequences, however.

A recent development in string theory has been the realization that as one moves away from exact Euclidean AdS [that is, hyperbolic, H^n] geometry — as of course one ultimately must [10] — various novel effects arise, and eventually one reaches geometries where specifically stringy effects [11] destabilize a spacetime *which may appear perfectly acceptable from a classical point of view.*

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[2]By contrast, the radius of the smallest spherical spatial slice of the simply connected global de Sitter spacetime is completely fixed by the cosmological constant.

For a concrete example, parametrise the three-torus by angles [taking a common minimal radius of 2K for the circles] and endow $\mathbb{R} \times T^3$ with the Euclidean metric

$$g_c(2,\, 2K,\, L)_{++++} \;=\; +\,\mathrm{d}t^2 \;+\; 4K^2 \cosh^2(\frac{t}{L})[\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2], \qquad (1)$$

where L is a constant; here, apart from the indication of the signature and the subscript c, which reminds us of the cosh function, the notation is as in [12][3]. Notice that, ignoring the signature, the "scale factor" is precisely that of de Sitter spacetime — only the geometry and topology of the spatial sections is different. The geometry has no obvious pathology; the manifold is non-singular and inextensible. For large t, we have

$$g_c(2,\, 2K,\, L)_{++++} \;\approx\; +\,\mathrm{d}t^2 \;+\; K^2\, e^{2\,t/L}\,[\mathrm{d}\theta_1^2 \;+\; \mathrm{d}\theta_2^2 \;+\; \mathrm{d}\theta_3^2]. \qquad (2)$$

The right side of this relation is the metric of constant curvature $-1/L^2$ on a partial compactification of H^4; it is precisely a four-dimensional version of the metric on the partially compactified two-dimensional hyperbolic space discussed by Ooguri et al [1]. Thus we can think of the geometry corresponding to $g_c(2,\, 2K,\, L)_{++++}$ as being obtained by deforming a certain quotient of ordinary Euclidean AdS space. Near infinity, the two spaces are almost indistinguishable geometrically. Physically, however, they are very different, as we shall see.

The difference is revealed by the behaviour of branes on these spaces. Let us introduce a 4-form field on these backgrounds, and consider the nucleation of Euclidean BPS 2-branes [13]. The stability of this system is determined by a *purely geometric* question: can the area of a brane always grow quickly enough to keep the action positive? In ordinary [non-compactified] Euclidean AdS it does, but in certain distorted versions of AdS it does not; thus we obtain a criterion for the stringy stability of the Euclidean version of a given spacetime. This Seiberg-Witten [13] mechanism has been applied to topologically non-trivial black hole spacetimes in [14]. It has recently been applied to *cosmology* by Maldacena and Maoz [15], and their work has been extended in various ways in [16][17][18][19][11][9]. [Other uses of instability to remove candidate backgrounds may be found in, for example, [20].]

For spaces [like those represented by the metrics in (1) and (2) above] which are *flat* at infinity this question of stability is particularly subtle [11]; the system can be stable or unstable, depending in a delicate way on the precise details of the geometry. A direct computation [12] shows that the brane action on the background given by (1) is in fact unbounded below, while it is always positive for the metric on the right side of (2): the volume term "wins" in the former case but not in the latter, despite the close similarity of (1) and (2) when t is large[4]. Thus the innocent appearance of (1) is very misleading. This is not a consistent background for string theory. On the other hand, the four-dimensional OVV space with metric (2) is consistent in this sense, despite the fact that its difference from $g_c(2,\, 2K,\, L)_{++++}$ decays rapidly towards infinity. Nevertheless, the OVV space is "close" to being unstable: although it is locally the same as Euclidean AdS_4, its global structure is different, and this affects the rate at which the area and volume of a brane can grow.

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[3]If this is a metric for a string gas cosmology, or if one wants to use it in the Hartle-Hawking manner, one should restrict t to $t \geq 0$ in this formula.

[4]That is, the approximation in (2) is not good enough in this context.

This conclusion is somewhat disturbing, for it might well apply to the kind of cosmological models one hopes to derive from string theory. One does not expect de Sitter spacetime itself to suffer from such instabilities — but we do not live in de Sitter spacetime: we live in a version of it that has been distorted by the presence of matter and radiation. The danger is that this distortion might have the same destabilizing effects as the one which turns the stable OVV space into the unstable space with metric (1). Notice, in particular, that our spacetime has been distorted away from de Sitter spacetime to this extent: it appears to be spatially flat, while de Sitter spacetime has spherical sections. Our Universe may well, therefore, have a conformal infinity which, as in the space with metric (1), is flat and compact. In view of the fact that the flat-boundary case is so delicate, *an apparently realistic string cosmology could be perilously close to being non-perturbatively unstable.* Clarifying this question is one of our major objectives.

We shall proceed as follows: first we discuss the unfamiliar way in which the space considered by Ooguri et al [1], and its four-dimensional generalization, must be continued to Lorentzian signature. We explain in detail the claim of Ooguri et al that their *single* Euclidean space can continue to *two* distinct Lorentzian spacetimes: this is possible with the aid of a slight extension of Witten's [21] technique of "multiple analytic continuation". The continuation can be performed in such a way that one obtains an accelerating cosmological spacetime ["OVV spacetime"] while avoiding the problem of a complex metric, mentioned in [1]. We then discuss the global structure of the OVV spacetime, stressing that it is null geodesically incomplete, though without curvature singularities.

Next, we show that a scalar field with a certain very simple potential can mimic the behaviour of "conventional" matter introduced into an initially OVV spacetime; here "conventional" means that the *Null Energy Condition* [NEC] is satisfied. The Einstein equations can be solved exactly when the matter equation-of-state parameter is constant, and so we can check directly that the brane action discussed above remains positive throughout the corresponding Euclidean space. However, this raises another problem, for, precisely when the NEC is satisfied, this spacetime has to have a curvature singularity if the Einstein equations hold, *even if the usual Strong Energy Condition is violated.* Thus we are in a paradoxical situation: the introduction of conventional matter stabilizes the spacetime and yet renders it singular. The only way out is to modify the geometry of the very early Universe in some way that does not induce Seiberg-Witten instability. This is a delicate matter, since such modifications, if they last too long, certainly do cause instability: indeed, this is why the space with metric $g_c(2, 2K, L)_{++++}$ is unstable.

The use of a scalar field to mimic the effects of matter and radiation has the further benefit that it permits a simple check of stability from a holographic point of view. Scalar fields on asymptotically de Sitter spacetimes induce a conformal field theory at infinity [22][23]; this is independent of the hypothetical existence of a complete "dS/CFT correspondence". This CFT frequently has complex conformal weights, which may possibly signal yet another instability or other pathology. We show that this does not happen in our model.

Finally, again guided by [1], we shall discuss how the singularities mentioned above can be resolved. This necessarily involves a brief violation of the *Null Ricci Condition* [NRC]. The resulting overall structure emphasises the link with string gas cosmology [7][12].

2. Euclid to Lorentz According to Ooguri et al

Ooguri et al work with a compactification of IIB string theory on a Euclidean manifold of the form $\mathbb{R} \times S^1 \times S^2 \times CY$, where CY is a Calabi-Yau manifold, where S^2 has its usual metric of curvature $1/L^2$, and where $\mathbb{R} \times S^1$ is a *partial compactification* of the hyperbolic surface of curvature $-1/L^2$. Recall that one form of the metric on the simply connected hyperbolic surface H^2 [topology $\mathbb{R}^2$] is

$$g(H^2)_{++} = e^{(2\zeta/L)} d\varrho^2 + d\zeta^2, \tag{3}$$

where both ϱ and ζ range in $(-\infty, +\infty)$. Ooguri et al work with the space [of topology $\mathbb{R} \times S^1$ and geometry $H^2/\mathbb{Z}$] obtained by compactifying ϱ but *not* ζ.

Leaving aside the $CY \times S^2$ factor for the moment, we have here a two-dimensional Euclidean space which Ooguri et al interpret in *two different ways*. In one, the circular version of ϱ is "time", leading to a Witten index which counts ground states of a certain black hole configuration. In the other, the circular coordinate is "space", and we have a cosmological model, which Ooguri et al regard as evolving in Euclidean "time", ζ, from an "initial" S^1. As ζ increases, the size of the "spatial" circle increases exponentially, so, physically, what we should obtain in this way is an *accelerating cosmology*. Of course, in speaking of "time" and "space", we are really referring to some Lorentzian version of this Euclidean space, so what Ooguri et al are proposing is that one has a single Euclidean space with *two different Lorentzian versions*, one of which is a two-dimensional accelerating cosmology. One of our objectives will be to clarify how it is possible for a single Euclidean space to have more than one Lorentzian version.

Meanwhile, however, let us note that Ooguri et al are really proposing a kind of *duality* between supersymmetric black holes and accelerating cosmologies. If this can be made to work, then we will have *a new tool for studying accelerating cosmologies* from a string-theoretic point of view, assuming that such cosmologies can be obtained in string theory in some other way [24]. The procedure would be to extend from two to four dimensions by replacing $H^2/\mathbb{Z}$ with its natural higher-dimensional version, the partial compactification $H^4/\mathbb{Z}^3$ [obtained by compactifying the sections in the foliation of H^4 by flat slices]; the most obvious replacement for $\mathbb{R} \times S^1 \times S^2 \times CY$ would then be $\mathbb{R} \times T^3 \times FR$, where $\mathbb{R} \times T^3$ is the topology of $H^4/\mathbb{Z}^3$, and where FR denotes a [singular] Freund-Rubin space of the kind studied in [25][26][27].

This approach differs from that of Maldacena and Maoz [15], who have constructed string-motivated cosmological models by introducing matter into locally *anti-de Sitter* spacetimes. These cosmologies can be made to accelerate [temporarily], but only by introducing a quintessence field [17][5].

However, this is not what Ooguri et al have in mind. They hope to obtain a direct interpretation of the $H^2/\mathbb{Z}$ metric *itself* in terms of a cosmology which accelerates without the aid of any quintessence field. Clearly this will only be possible by means of some unusual mathematical approach, because one normally associates hyperbolic metrics [constant *negative* curvature] with anti-de Sitter spacetime rather than de Sitter spacetime. Furthermore, as Ooguri et al themselves observe, it is far from clear how to interpret ζ

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[5]See [28][18][9][29] for other studies of anti-de Sitter cosmology.

in (3) as Lorentzian time, because complexifying it in the traditional way will complexify the metric. These two problems will in fact prove to be two sides of the same coin.

The key idea here is that of *multiple analytic continuation*, as used in studies of "bubble of nothing" spacetimes. The idea [originally suggested in [21]] is that it can be useful and physically meaningful to complexify *spacelike* as well as timelike dimensions of a Lorentzian spacetime, provided that in the end one obtains a real metric with physically reasonable signature. Thus, for example, it is acceptable to complexify both a time and a space coordinate in certain black hole and other solutions [30][31][32] to obtain other interesting spacetimes ["bubbles of nothing"]. Applied to the Euclidean case, this clearly opens the way to obtaining two [or more] different Lorentzian spacetimes from a single Euclidean space, just as Ooguri et al require. To understand how this works for asymptotically de Sitter and anti-de Sitter spacetimes, let us first explain how it applies to dS and AdS themselves. After doing so, we shall return to the specific case of the OVV spacetime at the end of this section.

It is a basic fact that if one takes the standard metric on the four-sphere of radius L,

$$g(\mathrm{S}^4)_{++++} = \mathrm{L}^2 \left\{ \mathrm{d}\xi^2 + \cos^2(\xi) \left[\mathrm{d}\chi^2 + \sin^2(\chi) \{ \mathrm{d}\theta^2 + \sin^2(\theta) \, \mathrm{d}\phi^2 \} \right] \right\}, \qquad (4)$$

where all of the coordinates are angular, and continues $\xi \to \mathrm{iT}/\mathrm{L}$, then the result is de Sitter spacetime,

$$g(\mathrm{dS}_4)_{-+++} = -\mathrm{dT}^2 + \mathrm{L}^2 \cosh^2(\mathrm{T}/\mathrm{L}) \left[\mathrm{d}\chi^2 + \sin^2(\chi) \{ \mathrm{d}\theta^2 + \sin^2(\theta) \, \mathrm{d}\phi^2 \} \right], \qquad (5)$$

with the indicated signature.

Now let us begin again, applying the ideas of "multiple analytic continuation" [21]. Let us complexify χ in equation (4) instead of ξ, replacing $\chi \to \pm \mathrm{is}/\mathrm{L}$, and for convenience relabelling ξ as u/L [without complexifying it]. This is a multiple analytic continuation in Witten's [21] sense, that is, it changes the sign of *more than one* term in the metric. We obtain

$$g(\mathrm{AdS}_4)_{+---} = \mathrm{du}^2 - \cos^2(\mathrm{u}/\mathrm{L}) \left[\mathrm{ds}^2 + \mathrm{L}^2 \sinh^2(\mathrm{s}/\mathrm{L}) \{ \mathrm{d}\theta^2 + \sin^2(\theta) \, \mathrm{d}\phi^2 \} \right]. \qquad (6)$$

But this is the anti-de Sitter metric, in $(+\ -\ -\ -)$ signature, and expressed in terms of coordinates [33][34][35] based on the timelike geodesics which are perpendicular to the spatial sections; the coordinate u is proper time along these geodesics. These coordinates do not cover the entire spacetime, of course, because these timelike geodesics intersect, being drawn together by the attractive nature of gravity in anti-de Sitter spacetime [which satisfies the Strong Energy Condition]. This is why these coordinates give the false impression that there is no timelike Killing vector in this geometry. In fact, these coordinates cover the Cauchy development of a single spacelike slice. Thus we have only continued S^4 to a small *part* of AdS_4, so the procedure is unsatisfactory in this sense; in particular, in the opposite direction, this way of Euclideanizing AdS_4 would obviously not be suitable for describing its asymptotic regions. Nevertheless it is now clear that it is not correct to claim that de Sitter spacetime is *the only* Lorentzian continuation of the four-sphere: one can even obtain a piece of anti-de Sitter spacetime in this way. In fact, it is clear more generally that Euclidean spaces will often have more than one Lorentzian version if we

accept both $(+ - - -)$ and $(- + + +)$ signatures — as we must, since the difference is a mere matter of convention[6].

For many purposes, the more familiar $\xi \to iT/L$ complexification of S^4 is the most satisfactory way of thinking about Euclidean de Sitter spacetime; one might well prefer it when considering the *earliest* history of the Universe. However, there is one very crucial aspect of de Sitter spacetime that is *not* well described in this way: its asymptotic region. Just as complexifying AdS_4 to S^4 is not a good way to study the boundary physics of anti-de Sitter spacetime, so also we cannot expect to understand the *late-time*, near-boundary physics of dS_4 by continuing it to S^4, which of course has no boundary whatever. This argument suggests strongly that a full understanding of asymptotically de Sitter spacetimes can only be obtained by *complementing* the standard Euclidean version of de Sitter spacetime with a continuation to a Euclidean manifold which, unlike S^4, can be regarded as the interior of a manifold-with-boundary. One picture would be suitable for very *early* times, the other for very *late* times.

The solution of this problem is to regard de Sitter spacetime as the continuation not only of S^4, *but also of the hyperbolic manifold* H^4. This was proposed [in this context] in [37]; explicit implementations were investigated in [38] and [39] [see also [40]]; it has been used in the very interesting theory of Lasenby and Doran [41][42]; it has been put on a rigorous mathematical basis [though mainly in the case of Einstein bulks, which are of limited cosmological interest] by Anderson [43]; closely related ideas appear in the theory of S-brane and "bubble" spacetimes [32]; it is relevant to any theory which makes use of the fact that the de Sitter and anti-de Sitter spacetimes are mutually locally conformal [44]; and, as we shall see, it plays a basic role in the recent work of Ooguri, Vafa, and Verlinde [1]. Let us explain why this suggestion is not as radical as it may seem.

First, one is [now] accustomed to think of dS_4 as a space of constant *positive* curvature, and hence it seems "natural" to associate it with S^4. However, this is merely due to the [current] preference for signature $(- + + +)$: if we use $(+ - - -)$ signature, then the curvature of dS_4 is *negative*. Similarly the curvature of the AdS_4 metric in $(+ - - -)$ signature, given in (6) above, is *positive*, as it should be since (6) was obtained from the metric of the Euclidean four-sphere. Again, the overall sign is a matter of convention, so there is no basis for the assertion that de Sitter spacetime is "naturally" associated with S^4. Indeed, in some works [for example [41][42]] the opposite is assumed to be natural.

A more serious objection is that AdS_4 can of course *also* be obtained by analytic continuation from H^4. Now AdS_4 differs from dS_4 not just in its local geometry but also in its global structure. How can these different global structures arise from different complexifications of a single Euclidean space?

First let us obtain the formal derivation. One global foliation of H^4, shown in Figure 1, is in terms of cylinders with topology $\mathbb{R} \times S^2$. This works as follows. H^4 can be defined as a connected component of the locus

$$-A^2 + B^2 + x^2 + y^2 + z^2 = -L^2, \tag{7}$$

defined in a five-dimensional Minkowski space. It is clear that all of the coordinates except A can range in $(-\infty, +\infty)$, while A has to satisfy $A^2 \geq L^2$. Choosing the connected

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[6]It is in fact possible to dispute this statement in the case of non-orientable spacetimes [36], but we shall not consider this case here.
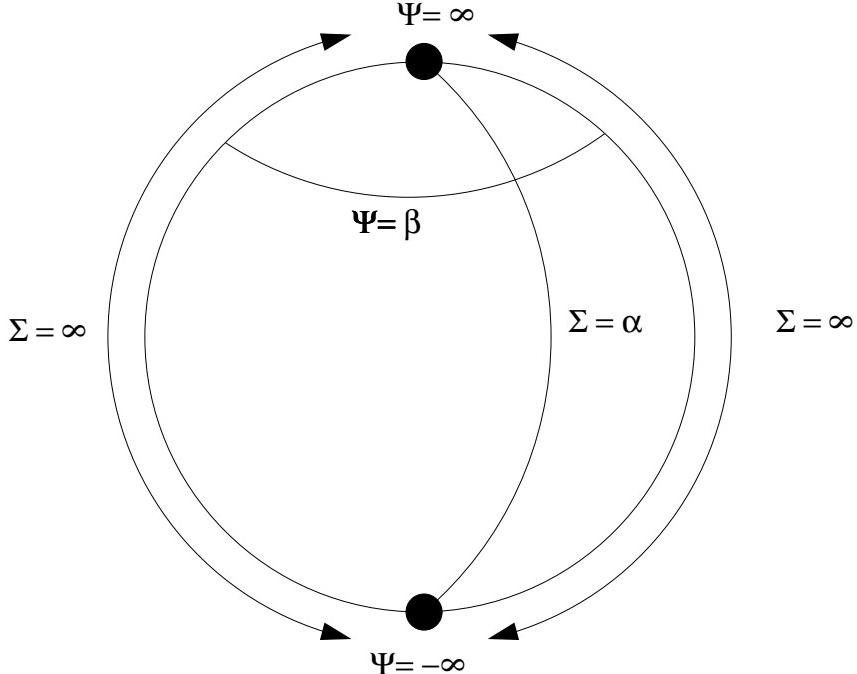
Figure 1: Cylindrical foliation of H^4.

component on which A is positive, we can pick coordinates Ψ,Σ,θ,ϕ such that

$$
\begin{aligned}
A &= L\cosh(\Psi)\cosh(\Sigma) \\
B &= L\sinh(\Psi)\cosh(\Sigma) \\
z &= L\sinh(\Sigma)\cos(\theta) \\
y &= L\sinh(\Sigma)\sin(\theta)\cos(\phi) \\
x &= L\sinh(\Sigma)\sin(\theta)\sin(\phi),
\end{aligned}
\tag{8}
$$

and these coordinates cover H^4 globally if we let Ψ run from $-\infty$ to $+\infty$ while Σ runs from 0 to $+\infty$. [Both θ and ϕ are suppressed in Figure 1.] The surfaces $\Psi = $ constant are copies of three-dimensional hyperbolic space, *all of the same curvature*, $-1/L^2$, as can be seen by noting that the first two equations of (8) imply that $-A^2 + B^2$ is independent of Ψ. These slices intersect the conformal boundary at right angles. A typical surface $\Psi = \beta = $ constant is shown in Figure 1.

The surfaces $\Sigma = $ constant are topological cylinders. If we think of H^4 as the interior of a four-dimensional ball, then these cylinders are "pinched" as they approach the boundary at the points $\Psi = \pm\infty$. A typical "pinched cylinder" inside the boundary is shown as $\Sigma = \alpha = $ constant in Figure 1. It is clear that H^4 is completely foliated by these cylinders, and the conformal boundary is a cylinder with two additional points [corresponding to $\Psi = \pm\infty$] added: with these additions, the boundary becomes the familiar three-sphere. Notice that this structure is very similar to that of Lorentzian anti-de Sitter spacetime. The metric with respect to this foliation is

$$
g(H^4)_{++++} = L^2\left\{\cosh^2(\Sigma)\,d\Psi^2 + d\Sigma^2 + \sinh^2(\Sigma)[d\theta^2 + \sin^2(\theta)d\phi^2]\right\}.
\tag{9}
$$

Now we shall consider a second, completely different, but also entirely global foliation of H^4, shown in Figure 2. Choose coordinates Θ,ρ,θ,ϕ, where Θ runs from $-\infty$ to $+\infty$
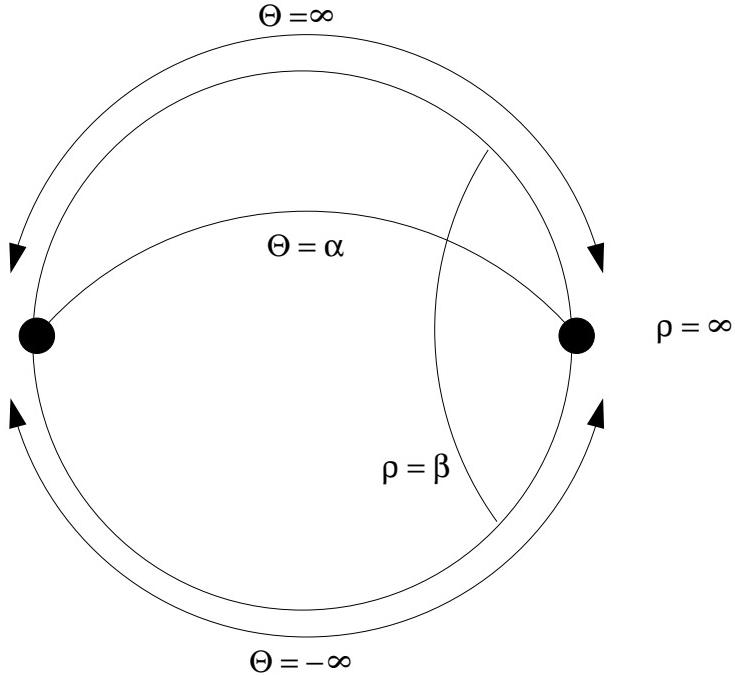
Figure 2: H^3 foliation of H^4.

while ρ runs from 0 to $+\infty$, and set

$$
\begin{aligned}
A &= L\cosh(\Theta)\cosh(\rho) \\
B &= L\sinh(\Theta) \\
z &= L\cosh(\Theta)\sinh(\rho)\cos(\theta) \\
y &= L\cosh(\Theta)\sinh(\rho)\sin(\theta)\cos(\phi) \\
x &= L\cosh(\Theta)\sinh(\rho)\sin(\theta)\sin(\phi).
\end{aligned}
\tag{10}
$$

Because we are suppressing two angles, Figure 2 seems to resemble Figure 1, but this is misleading [except in two dimensions, see below]. Here the surfaces $\Theta = $ constant are, from the second member of equations (10), just the submanifolds $B = $ constant; they are copies of the three-dimensional hyperbolic space H^3, as can be seen at once from equation (7). This foliation differs from the previous one in a crucial way, however: whereas previously the slices all had the same curvature, $-1/L^2$, as the ambient space, here the surface $\Theta = \alpha = $ constant can be written as

$$
-A^2 + x^2 + y^2 + z^2 = -L^2\cosh^2(\alpha),
\tag{11}
$$

so the magnitude of the curvature of a slice is reduced by a factor of $\operatorname{sech}^2(\alpha)$. The slices become flatter as they are expanded towards the boundary. In this case, the copies of H^3 are all "pinched together" as we move towards *their* boundaries, that is, as $\rho \to \infty$. A typical H^3 slice, $\Theta = \alpha = $ constant is shown in Figure 2.

Notice that the slices themselves do not intersect: only their conformal completions do so. At any point on a given copy of H^3, one can send a geodesic [shown in Figure 2] of the form $\theta = \phi = $ constant, $\rho = $ constant $= \beta$, towards infinity, and this will uniquely define two points on the boundary, one each at $\Theta = \pm\infty$. In this sense, one can

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say that conformal infinity is "disconnected": the usual three-sphere is divided into two hemispheres corresponding to the forward or backward "evolution" of any H^3 slice along Θ. Of course, topologically the boundary is connected, since the two hemispheres join along the common conformal boundary of all of the slices. Nevertheless, it will be useful to remember that the boundary does fall into two disconnected pieces if the boundary of the slices is deleted.

It is clear that this foliation is also global, though it is in general totally different to the one shown in Figure 1. The metric with respect to this foliation is

$$g(H^4)_{++++} \; = \; L^2 \left\{ d\Theta^2 \; + \; \cosh^2(\Theta)[d\rho^2 \; + \; \sinh^2(\rho)\{d\theta^2 \; + \; \sin^2(\theta)\,d\phi^2\}] \right\}. \qquad (12)$$

[See for example [45] for a different application of this foliation of H^4.]

Now the metrics in (9) and (12) are one and the same; only the foliations are different. It has been stressed by Buchel and Tseytlin [46][16], however, that it is not the case that two distinct foliations of a given Euclidean space must be regarded as fully equivalent in the quantum theory. This is true even in the case where different foliations correspond, on the Lorentzian side, to different foliations of the spacetime by spacelike slices with distinct intrinsic geometries. For this will correspond to distinct time evolutions with physically distinct quantum Hamiltonians. [On the Lorentzian side, different spacetime foliations correspond to different groups of observers with non-intersecting worldlines, so one should think of this in terms of observer dependence rather than violation of diffeomorphism invariance.] One should think of "Euclideanization" as an assignment to a given Lorentzian manifold of a Euclidean manifold *with a fixed foliation.* In our case the distinction between the two foliations pictured is even more stark, since, after complexification, the leaves of one foliation become spacelike while those of the other are themselves Lorentzian. Thus we should not in general regard Figures 1 and 2 as depictions of the same physical system, even though the underlying Euclidean manifold is the same in both cases.
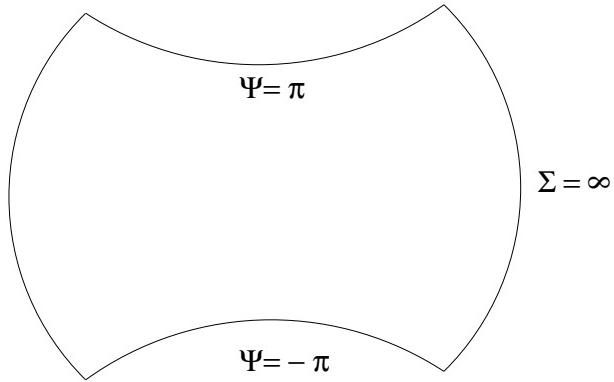

Figure 3: Cylindrical foliation of H^4, with compactification of one axis.

Furthermore, different foliations of a given Euclidean space can be appropriate to different ways of taking quotients by discrete isometry groups. For example, it is quite natural to compactify the coordinate Ψ in equation (9), since $\Psi \; \to \; \Psi \; + \; 2\pi$ is clearly an isometry. If we do this, then Figure 1 has to be re-drawn; parts of the diagram have to be deleted, as shown in Figure 3, and the top and bottom of the diagram have to be

identified. The boundary changes topology from S^3 to $S^1 \times S^2$. This is just the "thermal AdS" construction which is of such importance in the AdS/CFT correspondence [10]. By contrast, in (12) neither Θ nor ρ can be compactified in this way, at least not in four dimensions [see below]. Hence this foliation is appropriate to the case where we do *not* take the quotient. However, the *local* geometry of the two, now distinct, Euclidean spaces remains identical.

Let us now proceed to the complexifications. For the sake of clarity we shall adopt the convention that a non-periodic *time* coordinate becomes periodic, and vice versa, upon complexification; note that this is compatible both with our discussion above and with the usual procedure when continuing, for example, the Schwarzschild metric. There is however no reason to insist on this rule when dealing with *spacelike* coordinates. In that case we shall try to preserve the periodicity or non-periodicity of the coordinate, unless the geometry is such that it seems natural to do otherwise.

In view of our earlier discussion, we should not expect (9) and (12) necessarily to lead to the same Lorentzian spacetime upon complexification. Indeed, if we map $\Psi \to iU/L$ and re-label Σ [without complexifying it] as S/L, then we obtain from (9)

$$g(\mathrm{AdS}_4)_{-+++} \;=\; -\cosh^2(\mathrm{S/L})\,\mathrm{dU}^2 \;+\; \mathrm{dS}^2 \;+\; \mathrm{L}^2\sinh^2(\mathrm{S/L})[\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\mathrm{d}\phi^2], \quad (13)$$

and this is precisely [33] the *globally valid* AdS_4 metric, in the indicated signature[7]. But if we take (12) and perform a multiple analytic continuation [again, in Witten's sense [21] of changing the sign of more than one term in the metric] by mapping $\rho \to \pm i\chi$ while re-labelling Θ as T/L, we obtain, since $\sinh(\pm\,i\,\chi) = \pm i\sin(\chi)$,

$$g(\mathrm{dS}_4)_{+---} \;=\; \mathrm{dT}^2 \;-\; \mathrm{L}^2\cosh^2(\mathrm{T/L})\,[\mathrm{d}\chi^2 \;+\; \sin^2(\chi)\{\mathrm{d}\theta^2 \;+\; \sin^2(\theta)\,\mathrm{d}\phi^2\}], \quad (14)$$

which is of course the *global* form of the de Sitter metric, but in a signature which makes its curvature *negative*. Thus de Sitter and anti-de Sitter spacetimes are seen to have a common origin in the same Euclidean space, which has however been foliated in two ways that, in four dimensions, are very different[8].

Notice that, just as the cylindrical structure of the foliation in Figure 1 is very much like the cylindrical structure of Lorentzian anti-de Sitter spacetime, so also the structure of Figure 2 is very closely related to that of Lorentzian de Sitter spacetime, in the following sense: at each point of a spacelike section of Lorentzian de Sitter spacetime, one can send out a timelike geodesic either to the future or the past, and so map the point to a unique point on either future or past conformal infinity. We saw that a precisely analogous statement could be made regarding Figure 2. The difference is that the conformal infinity of Lorentzian de Sitter spacetime is truly disconnected, whereas the Euclidean "future" and "past" in Figure 2 bend around and touch along the equator, so that the full conformal infinity is one connected copy of S^3. If one were to delete this equator then the formal similarity would be still closer.

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[7]Unless we take the universal cover, the topology of AdS_4 is $S^1 \times {I\!\!R}^3$, so the time coordinate U is, in the first instance, a circular coordinate; see [33][34][35]. According to our rule, we pass to the universal cover in the Lorentzian case if and only if Ψ is periodic.

[8]If we compactify Ψ then we should say that they come from Euclidean spaces with the same *metric* but with different *topologies*.

We are not of course claiming that these simple observations immediately allow a full understanding of de Sitter spacetime in terms of [say] the Euclidean AdS/CFT correspondence. Our original motivation [37] for relating de Sitter spacetime to hyperbolic space was the observation that, from the point of view of symmetries, H^4 is much more similar to dS_4 than to AdS_4. Indeed, H^4 and dS_4 have exactly the same isometry group, the orthogonal group O(1,4). Furthermore, the conformal boundary of dS_4 consists of two copies of S^3, which of course is also the conformal boundary of H^4. These facts led us, in [37], to propose that one or both of the boundary components of dS_4 should be topologically identified with the boundary of H^4, thus enabling the symmetries of one side to act directly on the other. A concrete proposal for relating the physics on one side to that on the other was then proposed in [38] [see also[40]]. The relationship is non-local, and the transformation has a non-trivial kernel, so that information is lost as one tries to transfer de Sitter data to H^4. Thus one cannot establish an *exact equivalence* of the AdS/CFT kind here. Nevertheless it is certainly reasonable to expect that gross features of the physics on one side, such as the stability of an entire spacetime and its matter content, are reflected on the other.

All of our discussions thus far have been relevant to four-dimensional spacetimes. In other dimensions the situation is similar, though as we are about to see the case of two spacetime dimensions is rather subtle. However, the reader may wish to note the following point. Euclideanization of curved metrics was of course developed in connection with Euclidean Quantum Gravity [47], but the method has taken on a life of its own, particularly in connection with the AdS/CFT correspondence. Nevertheless, if one is interested in the original application, then it is important that complexification of a coordinate should also complexify the volume form. In equation (12), for example, the volume form is

$$\mathrm{dV}(g(\mathrm{H}^4)_{++++}) \;=\; \mathrm{L}^4 \cosh^3(\Theta) \sinh^2(\rho) \sin(\theta)\,\mathrm{d}\Theta\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi, \qquad (15)$$

and one sees at once that complexifying ρ [to obtain equation (14)] does indeed complexify the volume form. However, this only works if the number of spacetime dimensions is *even*. Thus we shall confine ourselves to even spacetime dimensions henceforth. [Depending on the dimension, one may have to choose the sign of the imaginary factor in the complexification so that the volume form "rotates" in the correct direction. This is the reason for the $\pm$ sign in the complexification of ρ, above.]

In two dimensions, the situation we have been describing is particularly interesting. In that case, Figures 1 and 2 can be interpreted literally, in the sense that there are no angles to be suppressed. Then a simple reflection, $\Psi \to \rho$, $\Sigma \to \Theta$ shows that the two foliations are identical, and indeed it is clear that the two ways of writing the Euclidean metric are the same:

$$g(\mathrm{H}^2)_{++} \;=\; \mathrm{L}^2\left[\cosh^2(\Sigma)\,\mathrm{d}\Psi^2 \;+\; \mathrm{d}\Sigma^2\right] \;=\; \mathrm{L}^2\left[\mathrm{d}\Theta^2 \;+\; \cosh^2(\Theta)\,\mathrm{d}\rho^2\right]; \qquad (16)$$

here, though not in higher dimensions, we can take ρ to be periodic, like Ψ. [We are therefore dealing with the quotient $\mathrm{H}^2/\mathbb{Z}$, and the correct Euclidean picture is then the one in Figure 3, or Figure 3 reflected about a diagonal if one prefers the other foliation.]

Complexifying as usual, we obtain

$$g(\mathrm{AdS}_2)_{-+} \;=\; -\cosh^2(\mathrm{S/L})\,\mathrm{dU}^2 \;+\; \mathrm{dS}^2, \qquad (17)$$

$$g(\mathrm{dS}_2)_{+-} \;=\; \mathrm{dT}^2 \;-\; \mathrm{L}^2 \cosh^2(\mathrm{T/L})\,\mathrm{d}\chi^2. \tag{18}$$

Now it is immediately clear that the two-dimensional case has a special property: *it is not possible to use the metric to decide which coordinate should be regarded as time.* For, locally, these two metrics are identical. Ultimately this is due to the fact that the anti-de Sitter group in n+1 dimensions, O(2,n), is isomorphic to the de Sitter group O(n+1,1), when n = 1. The local geometry does not tell us whether equation (17) is the AdS$_2$ metric with U as time, or (18) is the dS$_2$ metric with T as time [and a suitable choice of signature]. This corresponds precisely to the fact that the two Euclidean foliations are identical. However, our rule regarding the periodicity of time under complexification helps us: if we take Ψ to be periodic, then, if U is time, it should *not* be periodic: this gives us the standard topology $I\!\!R^2$ for two-dimensional anti-de Sitter spacetime. On the other hand, if χ is spacelike, then it should continue to be periodic, and this gives us the standard $I\!\!R \times \mathrm{S}^1$ topology for two-dimensional de Sitter spacetime. Thus the global structure helps to remove the ambiguity.

In this approach, the distinction between AdS$_2$ and dS$_2$ cannot be seen in the Euclidean theory: both spacetimes arise from the Euclidean space pictured in Figure 3. *The distinction arises from the choice as to which dimension is to be interpreted as time in the Lorentzian continuation.* Notice that neither interpretation involves complexifying the metric; nor does either lead to a periodic Lorentzian time: the periodicity is pushed off to Euclidean time in the AdS case, and to Lorentzian space in the dS case. This indicates that our rule as to when a complexified time coordinate should be periodic does have a firm physical basis.

In this work we shall be primarily concerned not with the de Sitter and anti-de Sitter spacetimes, but rather with spacetimes which are asymptotic to these fundamental examples. The above discussion was however necessary, because we must take care to perform analytic continuations in a way which is compatible with the continuation of the asymptotic spacetime. The basic point is that we now know how to continue an asymptotically hyperbolic Euclidean space to a cosmological spacetime. *This is precisely what is done in the work of Ooguri, Vafa, and Verlinde* [1], to which we now turn.

As we discussed earlier, the two-dimensional hyperbolic space can be foliated in yet a third way, by flat sections: the metric was given as (3). After compactifying one direction, as in [1], the manifold becomes $\mathrm{H}^2/Z\!\!\!Z$, and the metric becomes

$$g(\mathrm{H}^2/Z\!\!\!Z)_{++} \;=\; \mathrm{K}^2\, \mathrm{e}^{(2\,\zeta/\mathrm{L})}\,\mathrm{d}\vartheta^2 \;+\; \mathrm{d}\zeta^2; \tag{19}$$

here, ϑ as an angular coordinate. This partial compactification introduces a new length scale, denoted K, which is independent of the curvature scale L. This additional length scale is of basic importance.

As Ooguri et al point out, however, it is difficult to see how complexification can work in equation (19) if we proceed in the usual way, since complexifying ζ leads to a complex metric. Our earlier discussion reveals the correct procedure: if we think of ϑ as Euclidean "time" and complexify it as $\vartheta \to \mathrm{i}\tau/\mathrm{K}$ [re-labelling ζ as z], we obtain a *non-periodic* time coordinate τ on [part of] ordinary AdS$_2$:

$$g(\mathrm{AdS}_2^*)_{-+} \;=\; -\,\mathrm{e}^{2\mathrm{z}/\mathrm{L}}\,\mathrm{d}\tau^2 \;+\; \mathrm{dz}^2; \tag{20}$$

while if we think of K as a measure of the size of Euclidean "space", then the continuation K $\rightarrow \pm$iK, with a re-labelling of ζ as t and of ϑ as θ_1, gives us a local de Sitter spacetime:

$$g(\mathrm{dS}_2^*/\mathbb{Z})_{+-} \; = \; \mathrm{dt}^2 \; - \; \mathrm{K}^2 \, \mathrm{e}^{2\mathrm{t}/\mathrm{L}} \, \mathrm{d}\theta_1^2. \tag{21}$$

Here the asterisks remind us that the foliations of *Lorentzian* AdS and dS by flat sections do not cover the entire spacetimes. Furthermore, these flat sections are not compact in the original spacetime, so if [in accordance with our rule] the coordinate θ_1 is periodic, then we are dealing with $\mathrm{dS}_2^*/\mathbb{Z}$ here, as indicated. This space happens to have the same $\mathbb{R} \times \mathrm{S}^1$ topology as the standard two-dimensional de Sitter spacetime, but it is constructed quite differently: it is obtained[9] by cutting away part of dS_2, leaving a geodesically incomplete spacetime, and then patching this up by means of a topological identification. *The fact that one returns to the original topology in this way is merely a quirk of two-dimensional geometry.* This observation will be crucial below; meanwhile, we stress that, while they may have the same metric, AdS_2^* and $\mathrm{dS}_2^*/\mathbb{Z}$ are globally very different. This is how one distinguishes the two different Lorentzian spacetimes which Ooguri et al wish to obtain from a single Euclidean space.

We have obtained a consistent picture of the proposal of Ooguri et al, *one which avoids the problem of complex metrics.* The analysis also instructs us how to work towards more realistic, four-dimensional cosmological models. We should begin with the four-dimensional version of the Euclidean space $\mathrm{H}^2/\mathbb{Z}$, namely $\mathrm{H}^4/\mathbb{Z}^3$, where, as always, H^n denotes the standard simply connected space of constant negative curvature $-1/\mathrm{L}^2$; here, we take the H^4 foliation by simply connected flat surfaces, as described for example in [6], and compactify these surfaces as we did in the two-dimensional case. The simplest choice is a cubic torus with all circumferences equal to position-dependent multiples of $2\pi\mathrm{K}$; other shapes and topologies are possible and interesting but will not be considered here. With this construction we obtain the manifold $\mathbb{R} \times \mathrm{T}^3$ with a metric which is just the obvious extension of $g(\mathrm{H}^2/\mathbb{Z})_{++}$ [equation (19) above]. This is a space of constant negative curvature $-1/\mathrm{L}^2$; it is a partial compactification of H^4. Therefore one can study in it all of the usual Euclidean AdS physics, such as the Seiberg-Witten [13] brane pair creation process.

We are interested here in the cosmological Lorentzian version of the "Euclidean OVV space", $\mathrm{H}^4/\mathbb{Z}^3$. In accordance with the above discussion, we should therefore complexify K in the same way as in the two-dimensional case; since this will change the signature from $(+ \; + \; + \; +)$ to $(+ \; - \; - \; -)$ [because K is the radius of all three circles], this can be regarded as another example of Witten's [21] multiple analytic continuation. In fact, we *must* proceed in this way in order to ensure compatibility with the $(+ \; + \; + \; +)$ to $(+ \; - \; - \; -)$ complexification of the basic H^4 with which we began. We finally have the four-dimensional version of the OVV metric,

$$g_{\mathrm{OVV}}(\mathrm{K, L})_{+---} \; = \; \mathrm{dt}^2 \; - \; \mathrm{K}^2 \, \mathrm{e}^{(2\,\mathrm{t}/\mathrm{L})} \, [\mathrm{d}\theta_1^2 \; + \; \mathrm{d}\theta_2^2 \; + \; \mathrm{d}\theta_3^2]. \tag{22}$$

This is a Lorentzian metric with the indicated signature: thus it is still a metric of constant *negative* curvature, but it is *locally* indistinguishable from the $(+ \; - \; - \; -)$ version of dS_4 with its foliation by flat surfaces. As discussed earlier, complexifying K $\rightarrow \pm$ iK will

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[9]See the next Section for a discussion of this construction in the four-dimensional case.

complexify the volume form [which is proportional to K^n in (n+1) dimensions] provided that the number of spacetime dimensions is even; thus the path integral can be made to converge in the four-dimensional case if the sign of the imaginary factor is suitably chosen.

We have argued that, if the Ooguri et al proposals lead to a reasonable four-dimensional cosmology at all, then the basic form of this cosmology is a certain version of de Sitter spacetime, with the metric (22), and with a distinctive *topology*. Because of its fundamental importance, we need to study the properties of this spacetime in detail.

3. Structure and Stability of the OVV Spacetime

In this section we shall explore the very interesting global structure of the four-dimensional OVV spacetime, and we shall show that it is stable against the Seiberg-Witten pair production process.

First we consider the details of the construction. Consider the version of $(+ \; - \; - \; -)$ de Sitter spacetime with flat spatial sections. Physically, this is precisely the set of all events in dS_4 to which a signal can be sent by an inertial observer. This is in fact an open submanifold of $I\!R \times S^3$: the Penrose diagram [48] is obtained by deleting from the usual dS_4 diagram the lower triangular half, *including the diagonal and its endpoints*. This is shown as the triangle OAC in Figure 4; the fact that the endpoints of the diagonal have been deleted is indicated by the small circles. Notice that timelike geodesics perpendicular to the flat spatial sections are represented by straight lines converging on the point O. It is
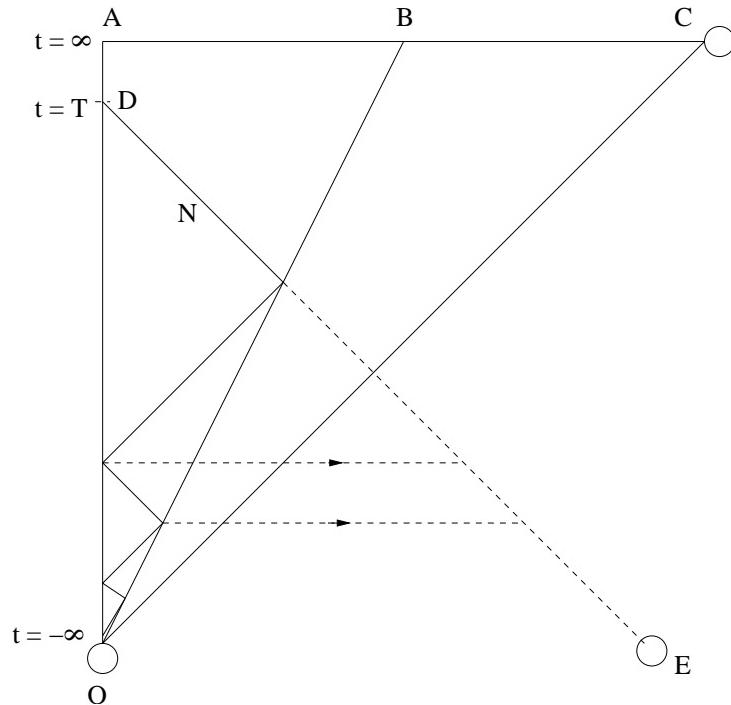
Figure 4: Penrose diagram of the OVV spacetime.

clear from the diagram that *this spacetime is timelike and null geodesically incomplete*, but we can *partly* remedy this by compactifying the spatial sections. This works as follows.

In the Penrose diagram, a typical point represents a two-sphere. We can however inscribe a cube into each two-sphere, with the cubes in different two-spheres having parallel faces, and then think of the points in the diagram in terms of the inscribed cubes. Since cubes do not of course have spherical symmetry, we will have to interpret the Penrose diagram as representing a specific direction. Now in the Penrose diagram of the half of de Sitter spacetime which can be foliated by copies of flat $\mathbb{R}^3$, take the cube of side length $2\pi K$ at time $t = 0$ and perform the usual identifications to generate a three-torus. The effect is to remove all of the diagram to the right of the line marked OB, which is the timelike geodesic corresponding to this cube. Thus the Penrose diagram corresponding to the OVV metric is just the triangle OAB shown, where however the point O is *not* included.

Clearly the line OB does not otherwise intersect the diagonal line OC where the original spacetime was incomplete, so the only place where the new spacetime can be incomplete is the point O. This point is at $t = -\infty$, so it is at an infinite proper time to the past along any timelike curve; thus this spacetime is timelike geodesically complete. However, consider the null curve N shown, which begins at $t = T$ and extends back to $t = -\infty$. Because of the topological identifications, it appears to "bounce" back and forth between the origin and OB: actually it wraps around the torus, which of course shrinks as we move back in time. It will in fact wrap around the torus infinitely many times. To compute the amount of affine parameter expended as it moves towards O, proceed as follows: first, write equation (22) as

$$g_{\text{OVV}}(K, L)_{+---} = e^{(2\,t/L)} \{d\eta^2 - K^2[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]\}, \tag{23}$$

where η is conformal time; thus $dt = e^{(t/L)}\,d\eta$. As the conformally related metric in the braces is flat, a null geodesic satisfies $d\eta/d\widetilde{\lambda} = c_1 = $ constant, where $\widetilde{\lambda}$ is an affine parameter with respect to this flat metric. The usual relation [[49], page 446] between the affine parameters of null geodesics of conformally related metrics gives us $d\widetilde{\lambda}/d\lambda = c_2\,e^{-(2\,t/L)}$, where c_2 is another constant. Thus we have

$$\frac{dt}{d\lambda} = \frac{dt}{d\eta}\frac{d\eta}{d\widetilde{\lambda}}\frac{d\widetilde{\lambda}}{d\lambda} = e^{(t/L)}\,c_1\,c_2\,e^{-(2\,t/L)} = c\,e^{-(t/L)}. \tag{24}$$

Now it should be clear from the construction of the line DE in Figure 4 that the "corners" in the null geodesic N are not relevant to the computation of the affine parameter A(N) along N. The answer is simply given by

$$A(N) = c \int_{-\infty}^{T} e^{(t/L)} dt, \tag{25}$$

which is obviously *finite*. Thus we have explicitly constructed a null geodesic which is inextensible and yet expends finite total affine parameter as it is extended back towards the point O. We conclude that the OVV spacetime is *timelike but not null geodesically complete*.

One might think that this incompleteness is a special property of the specific metric $g_{\text{OVV}}(K, L)_{+---}$, but this is *not* the case. In fact, a theorem[10] due to Andersson and

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[10]*Not* the one used in [12].

Galloway [50][51] implies that null incompleteness to the past is almost unavoidable here. The theorem may be stated as follows; we refer the reader to the Appendix for details of the terminology and a brief commentary on this remarkable result.

THEOREM [Andersson-Galloway]: *Let* M_{n+1}, $n \leq 7$, *be a globally hyperbolic (n+1)-dimensional spacetime with a regular future spacelike conformal boundary* Γ^+. *Suppose that the Null Ricci Condition is satisfied and that* Γ^+ *is compact and orientable. If* M_{n+1} *is past null geodesically complete, then the first homology group of* Γ^+, $H_1(\Gamma^+, \mathbb{Z})$, *is pure torsion.*

Here the *Null Ricci Condition* is the requirement that, for all null vectors k^μ, the Ricci tensor should satisfy

$$R_{\mu\nu} k^\mu k^\nu \geq 0. \qquad (26)$$

The OVV spacetime has a regular future conformal boundary, pictured as the line AB in Figure 4; this boundary is compact and orientable, since it is just the torus T^3. The Null Ricci Condition is satisfied, since the spacetime is an Einstein space. But the first homology group of T^3 is certainly *not* pure torsion [that is, not every element is of finite order]: it is isomorphic to $\mathbb{Z} \oplus \mathbb{Z} \oplus \mathbb{Z}$. Thus the spacetime *had* to be null geodesically incomplete.

The surprising and beautiful feature of the Andersson-Galloway theorem, however, is that this same conclusion holds *even if we distort the geometry*, as long as the Null Ricci Condition continues to hold. That is, if we introduce any kind of matter such that the NRC continues to hold — if the Einstein equations are valid, then this just means that the Null Energy Condition holds — then the resulting cosmology will necessarily be null incomplete to the past. We can interpret the theorem in this way because, physically, the introduction of matter into the OVV spacetime will not violate its other conditions. In other words, the Andersson-Galloway theorem is a *singularity theorem*: as with the classical singularity theorems, the conclusion does not depend on assumptions about local isotropy or homogeneity or on the precise structure of the Friedmann equations. The behaviour it predicts will occur no matter what happens to the spatial sections in the early Universe.

Obviously the null incompleteness of the OVV spacetime itself is rather harmless, since clearly there is no "singularity" at O. It is true that, in principle, an observer in this spacetime can receive a message from the "negative infinite" past; but since one can receive messages in ordinary AdS_4 from spatial infinity, perhaps this no longer seems very shocking. More seriously, however, one should note that this spacetime contains nothing but dark energy. If we were to introduce some ordinary matter or radiation satisfying the NRC, then we would certainly expect a real singularity to develop. The situation here is analogous to that of pure AdS_4, with metric given in equation (13). This space satisfies the *Strong* Energy Condition [SEC], and yet it is not singular. This does not contradict the Hawking-Penrose theorem, however, but only because AdS_4 does not satisfy the *generic condition* [[33], page 101], a technical condition on the curvature tensor. In the same way, the null geodesic incompleteness of the OVV spacetime will turn into a genuine curvature singularity if we introduce any kind of matter such that the Null Ricci Condition continues to hold, because the spacetime will become generic in this sense. This expectation is confirmed by the explicit examples we shall consider later.

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We stress that this result is *very much stronger* than the classical singularity theorems, which require the SEC. For it is easy to violate the SEC with well-behaved matter such as quintessence, but it is extremely difficult to do so in the case of the Null Energy Condition, a fact which has been discussed from many different points of view in the recent literature — see for example [52][53][54][12][55]. This is particularly true if any NEC violation persists beyond the earliest history of the Universe. We must nevertheless accept that any concrete cosmological model based on the OVV spacetime will be singular if we only consider non-exotic matter fields and if the Einstein equations hold exactly. This point will be discussed in more detail below.

Let us return to the OVV spacetime itself. Recall that the Penrose diagram is just the triangle OAB in Figure 4. Evidently, like de Sitter spacetime, the OVV spacetime has a cosmological horizon, which would extend diagonally down from point A in the Figure. Clearly, however, the observer at the origin can receive signals from any point in space[11]; indeed, as we saw, he can see rays of light which have circumnavigated his world arbitrarily many times. In reality, of course, he can see back no farther than the time of decoupling, but, even in this case, we can adjust K [which is inversely related to the slope of the line OB in Figure 4] so that he will *eventually* be able to see an entire spatial section even if — apparently like ourselves [58][59] — he cannot do so at present. At that time he will be able to deduce the fate of all physical systems in his world, despite the fact that they are destined ultimately to pass beyond the horizon. Thus the horizon has a rather different, less questionable status in this spacetime. See [60][12] for discussions of related issues.

Finally, let us consider the stability of this spacetime from the Seiberg-Witten [13] point of view. The fact that the Euclidean OVV space is locally indistinguishable from Euclidean AdS_4, which is of course stable in this sense, should not make us complacent. For branes, being extended objects, are sensitive to certain *global* geometric features: their actions typically involve non-local quantities, such as area and volume, the growth of which can be strongly affected by the global structure of the ambient manifold. One might be concerned that these global phenomena might affect area and volume differently. This does in fact happen in the OVV case, though fortunately not to the extent that any instability is induced.

Consider a BPS $(D-1)$-brane together with an appropriate antisymmetric tensor field in a Euclidean asymptotically AdS_{D+1} background. The brane action consists [13] of two terms: a positive one contributed by the brane tension, but also a *negative* one induced by the coupling to the antisymmetric tensor field. As the first term is proportional to the area of the brane, while the second is proportional to the volume enclosed by it, we have

$$S = T(A - \frac{D}{L}V),\tag{27}$$

where T is the tension, A is the area, V the volume enclosed, and L is the background asymptotic AdS radius. In the case at hand, [the Euclidean version of] equation (22) gives us

$$S_{OVV}(t) = 8\pi^3 T K^3 \left\{ e^{3t/L} - \frac{3}{L} \int_{-\infty}^{t} e^{3\tau/L} \, d\tau \right\},\tag{28}$$

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[11]See [56][57] for early discussions of such phenomena.

which of course is *precisely zero*. In reality, as we discussed above, this spacetime will become singular if we introduce non-exotic matter into it, so we should really begin the integration at some finite value, say $t = -\sigma$; then the action is the positive constant $8\pi^3 T K^3 e^{-3\sigma/L}$. The introduction of matter will however modify the geometry, and so the action will in general either increase or decrease away from this value as t is taken to infinity. If it becomes negative, then we have a non-perturbative instability of the system. Explicit examples of this have been given in [15] and [12].

Clearly the OVV spacetime is not unstable in this sense; however, it would not be difficult to render it unstable, since even a small constant negative contribution to the slope of the action could eventually lead to negative values for the action itself. Evidently we need to consider the back-reaction on the OVV spacetime geometry arising from the presence of matter. This is the subject of the next section.

4. Structure of OVV Cosmologies Containing Non-Exotic Matter

In this section we shall consider the consequences of introducing "non-exotic" matter into the OVV spacetime studied in the previous section. Here "non-exotic" means that the matter shall be such that the corresponding stress-energy tensor satisfies the Null Energy Condition [NEC]. *If* we assume that the Einstein equations hold, this is equivalent to assuming that the spacetime geometry satisfies the Null Ricci Condition[12]. However, the reader is reminded that in more general contexts, such as brane world models [63][64][65] or Gauss-Bonnet models [66], it is possible for the NRC to be violated even if all true matter fields satisfy the NEC. This is known as "effective" violation of the NEC; see [67] for a general discussion of this point. This will be important later, but, throughout this section, we shall assume that the Einstein equations do hold.

Our specific concern in this section is to understand the classical structure of the spacetimes obtained by deforming the OVV spacetime, using non-exotic matter. Questions of stability are postponed to the next section.

We proceed as follows. We take the OVV spacetime, with a small cosmological constant determined by a length scale L, and introduce into it a scalar field φ with a potential

$$V(\varphi,\,\epsilon) \;=\; -\,\frac{3}{8\pi L^2}\,[1\,-\,\frac{1}{6}\,\epsilon]\,\sinh^2(\sqrt{2\,\pi\,\epsilon}\,\varphi); \tag{29}$$

here ϵ is a positive constant. We have placed the minus sign prominently so as to remind the reader that, in the $(+\,-\,-\,-)$ signature being used here, the sign of a potential is the opposite of the familiar one. The kinetic term does not change sign, however. Bear in mind that OVV spacetime is *locally* the same as de Sitter spacetime, which in this signature has a negative cosmological constant $-3/(8\pi L^2)$.

While we shall also be interested in the specific behaviour of a small scalar field excitation on an OVV background, our primary concern at this point is with the following claim: we assert that, as far as the spacetime geometry is concerned, the field φ *exactly mimics* the effects on the OVV spacetime of a matter field with *constant* equation-of-state

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[12]See [61] for discussions of the observational aspects of NEC violation, and [62] for the theoretical aspects.

parameter related to ϵ by

$$\mathrm{w}_\varphi = \frac{1}{3}\epsilon - 1. \tag{30}$$

Thus for example if we insert non-relativistic matter [zero pressure] into the OVV space-time and allow it to act [via the Einstein equations], this will have the same effect as introducing φ with $\epsilon = 3$, while φ with the value $\epsilon = 4$ mimics the effects of radiation; the value $\epsilon = 1$ arises if we are interested in the back-reaction induced by domain walls on the OVV geometry, and so on. We stress that we are *not* primarily interested in using this field to violate the Strong Energy Condition; that is, φ is not [necessarily] a quintessence field. In the cases of principal interest to us, the acceleration is due to the negative contribution made by the OVV cosmological constant to the total pressure. Also note that we are *not* claiming that φ is a fundamental scalar field: we use it as a convenient way of representing various kinds of matter to be inserted into the OVV spacetime. If we take the kinetic term to be the standard one, then, as is well known, φ automatically satisfies the Null Energy Condition, so the condition that the matter should be "non-exotic" holds for any value of the parameters.

We now proceed to justify these claims. We shall consider Friedmann-like cosmological models with metrics of the form

$$g = \mathrm{dt}^2 - \mathrm{K}^2\,\mathrm{a(t)}^2[\mathrm{d}\theta_1^2 + \mathrm{d}\theta_2^2 + \mathrm{d}\theta_3^2]; \tag{31}$$

this generalizes the OVV metric in an obvious way. Adding the energy density of the φ field to that of the initial OVV space, we have a Friedmann equation of the form

$$(\frac{\dot{\mathrm{a}}}{\mathrm{a}})^2 = \frac{8\pi}{3}\,[\,\frac{1}{2}\,\dot{\varphi}^2 - \mathrm{V}(\varphi,\,\epsilon) + \frac{3}{8\pi\mathrm{L}^2}]. \tag{32}$$

The equation for φ itself is

$$\ddot{\varphi} + 3\,\frac{\dot{\mathrm{a}}}{\mathrm{a}}\,\dot{\varphi} - \frac{\mathrm{dV}(\varphi,\,\epsilon)}{\mathrm{d}\varphi} = 0. \tag{33}$$

Surprisingly, these equations have very simple solutions: by muddling about in the manner of [17], one finds that [with natural initial conditions] φ is given by

$$\varphi = \frac{1}{\sqrt{\pi\epsilon/2}}\tanh^{-1}(\mathrm{e}^{-\epsilon\mathrm{t}/2\mathrm{L}}), \tag{34}$$

and the metric is[13]

$$g_\mathrm{s}(\epsilon,\,\mathrm{K},\,\mathrm{L})_{+---} = \mathrm{dt}^2 - \mathrm{K}^2\,\sinh^{(4/\epsilon)}(\frac{\epsilon\,\mathrm{t}}{2\mathrm{L}})\,[\mathrm{d}\theta_1^2 + \mathrm{d}\theta_2^2 + \mathrm{d}\theta_3^2]; \tag{35}$$

here the s refers to the sinh function. From these results one can compute the energy density and pressure of the φ field alone:

$$\rho_\varphi = \frac{3}{8\pi\mathrm{L}^2}\,\mathrm{cosech}^2(\frac{\epsilon\,\mathrm{t}}{2\mathrm{L}}), \tag{36}$$

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[13]The reader who wishes to undertake the task of verifying these solutions will find the following simple fact helpful: if A and B are quantities related by $\tanh(\mathrm{A}) = \mathrm{e}^{-\mathrm{B}}$, then $\cosh(2\mathrm{A}) = \coth(\mathrm{B})$.

$$p_{\varphi} = \frac{3}{8\pi L^2} [\frac{1}{3}\epsilon - 1] \operatorname{cosech}^2(\frac{\epsilon t}{2L}), \tag{37}$$

from which equation (30) above is immediate.

If $\epsilon = 3$, we should have the local metric for a spacetime containing non-relativistic matter and a de Sitter cosmological constant, and indeed $g_s(\epsilon, K, L)_{+---}$ reduces in this case — purely locally, of course — to the classical Heckmann metric [see [68] for a recent discussion]. In the general case it agrees [again locally] with the results reported in [69], where it is obtained by postulating a linear equation of state [without giving a matter model]. For large t we have

$$g_s(\epsilon, 2^{2/\epsilon} K, L)_{+---} \approx dt^2 - K^2 e^{2t/L}[d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \tag{38}$$

which is the OVV metric given in equation (22); notice that ϵ effectively drops out. Thus our metric is "asymptotically OVV", for all ϵ.

The matter content of this spacetime does not behave as simply as one might expect. For while it is true that both components, the cosmological constant and the φ field, separately have constant equation-of-state parameters, *their combination does not*: if we denote the total energy density by ρ and the total pressure by p, then we have a total equation-of-state parameter w given by

$$w = p/\rho = -1 + \frac{\epsilon}{3}\operatorname{sech}^2(\frac{\epsilon t}{2L}). \tag{39}$$

Thus w decreases from $-1 + (\epsilon/3)$ in the early universe to its asymptotic OVV value -1. The Strong Energy Condition is violated if $w < -1/3$, so the SEC holds in the early universe provided that $-1 + (\epsilon/3) > -1/3$, which just means that ϵ should exceed 2. In this case there is a transition from deceleration to acceleration, as is observed in our Universe [70]. This is of course the case of most interest.

Clearly the field φ diverges as we trace it back towards t = 0, and so do its energy density and pressure. It follows from the Einstein equations that this spacetime has a genuine [curvature] singularity there: for example, the scalar curvature is given by

$$R(g_s(\epsilon, K, L)_{+---}) = -\frac{12}{L^2} + \frac{3}{L^2}(\epsilon - 4)\operatorname{cosech}^2(\frac{\epsilon t}{2L}); \tag{40}$$

this tends to $-12/L^2$ as t tends to infinity, the correct asymptotic de Sitter value in this signature, but it clearly diverges as t tends to zero [except in the $\epsilon = 4$ case, which corresponds to radiation and hence to a traceless stress-energy tensor which does not contribute to the scalar curvature; but this case is still singular, as one sees by examining other curvature invariants]. *This cannot be understood in terms of the classical singularity theorems*, because some of the metrics in this family violate the *Strong* Energy Condition even at early times, and yet remain singular: for example, the metric

$$g_s(2, K, L)_{+---} = dt^2 - K^2 \sinh^2(\frac{t}{L})[d\theta_1^2 + d\theta_2^2 + d\theta_3^2] \tag{41}$$

violates the SEC at *all* times t > 0, and yet it is singular. The same statement is true for all members of the family with ϵ less than 2. This is in contrast to spacetimes with the $\mathbb{R} \times S^3$ topology of [the global, simply connected version of] de Sitter spacetime,

where of course the classical singularity theorems can be evaded precisely because de Sitter spacetime itself violates the SEC. That is, SEC violation is enough to remove the singularity in that case, *but not here.*

Instead, we have to use the Andersson-Galloway theorem [50][51], which applies to these spacetimes in exactly the same way as we applied it to the OVV spacetime in the preceding section. That is, the failure of null geodesic completeness is an inevitable consequence of the *topology* of these spacetimes [the first homology group of the spatial sections is not pure torsion] combined with the fact that φ automatically satisfies the *Null* Energy Condition [which is equivalent to the Null Ricci Condition here since we are assuming the Einstein equations]. But, as we foresaw, the failure of null geodesic completeness here is more serious than in the pure OVV case: the spacetime curvature is no longer constant, the geometry is generic [in the technical sense discussed earlier], and the result is a curvature singularity, which enforces timelike as well as null geodesic incompleteness. The Andersson-Galloway theorem implies that this singularity can only be avoided in one way: by violating the NRC. This important conclusion will be discussed in more detail later.
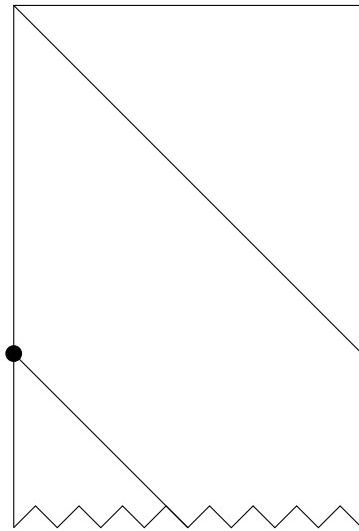
Figure 5: Penrose diagram of the $g_s(\epsilon, \text{K}, \text{L})_{+---}$ spacetime, $\epsilon > 2$

The complete structure of these spacetimes is summarized, as usual, in their Penrose diagrams, which are of two types according to the value of ϵ. The Penrose diagram for the $\epsilon > 2$ case is shown in Figure 5. Future conformal infinity is spacelike, and there is a Big Bang singularity which is also spacelike. The height/width ratio of the diagram is determined by the parameter ratio L/K; we have chosen a value such that the observer shown cannot *yet* detect the toral structure of his universe, though he will detect it later. The reader may find it helpful to compare Figure 5 with the diagrams in [60][71].

If $\epsilon \leq 2$, the Penrose diagram is triangular, like the triangle OAB in Figure 4, with the difference that the point O becomes genuinely singular. These diagrams and their precise shapes will be discussed in more detail elsewhere [72]. Meanwhile, we remind the reader that we are really most interested in values of ϵ between 3 [pure non-relativistic matter] and 4 [pure radiation].

We now turn to questions of stability.

5. Stability of OVV Cosmologies Containing Non-Exotic Matter

In this section we shall be concerned with the stability of these spacetimes, specifically from a holographic/string-theoretic point of view.

We have treated the φ field merely as a device for representing the effects of various kinds of matter and radiation on the OVV spacetime. However, let us ask what happens if we take it more seriously, as a genuine scalar field excitation propagating on the OVV background [without affecting the spacetime geometry].

The field theory of scalars on de Sitter-like backgrounds is a vast subject; see for example [73][74][75] for relevant work. A new aspect of the theory was revealed, however, by studies of de Sitter spacetime from a holographic point of view. It was soon realised [22][23] that a scalar field propagating in the de Sitter bulk induces a conformal field theory on the conformal boundary. Whether this implies the existence of a complete *equivalence* of the AdS/CFT kind is questionable [38][40], but we shall not rely on the existence of a complete "dS/CFT correspondence" here. We merely wish to ask what kind of CFT is induced by our field φ.

In detail, the limit of a scalar field amplitude [for a scalar of mass m_φ] in de Sitter spacetime defines a CFT two-point function at de Sitter conformal infinity. The conformal weights corresponding to the boundary operator defined by φ are given, in four spacetime dimensions, by

$$ h_\pm \;=\; \frac{1}{2}\,[3 \,\pm\, \sqrt{9 \,-\, 4\,L^2\,m_\varphi^2}]. \tag{42} $$

One sees immediately that the weights will be *complex*, with unwelcome physical consequences, unless m_φ satisfies $m_\varphi^2 \;\leq\; 9/(4L^2)$. This "Strominger bound" is analogous [22][37][39] to the well-known Breitenlohner-Freedman bound [76] on the masses of scalar fields in anti-de Sitter spacetime.

Now at late times, the metric $g_s(\epsilon, K, L)_{+---}$ [equation (35)] is locally indistinguishable from that of de Sitter spacetime, and, furthermore, φ is very small [equation (34)]; hence we see, from (29), that φ can be regarded as a scalar field of squared mass

$$ m_\varphi^2 \;=\; \frac{3}{2L^2}\,\epsilon\,[1 \,-\, \frac{1}{6}\,\epsilon] \tag{43} $$

propagating on a local de Sitter background. Substituting this into equation (42), we find that *the Strominger bound is automatically satisfied for all values of* ϵ. Both weights are always real, and they are given simply by

$$ \begin{aligned} h_+ \;&=\; \epsilon/2 \\ h_- \;&=\; 3 \,-\, (\epsilon/2); \end{aligned} \tag{44} $$

here we have assumed that $\epsilon \,\geq\, 3$ so that $h_+ \,\geq\, h_-$; of course the definitions should be reversed if $\epsilon \,<\, 3$. Notice that by varying ϵ one can obtain all values of the mass allowed by the Strominger bound. Recall that the action $V(\varphi, \epsilon)$ was constructed simply with a view to obtaining equation (30); so it is remarkable indeed that the construction is also precisely what is required to ensure that the Strominger bound is satisfied.

We conclude that the field φ always induces a well-behaved CFT at infinity; our matter models seem to be acceptable from a holographic point of view.

We complete this discussion with the following curious observation. One sees from (44) that the Strominger bound is saturated when $\epsilon = 3$. Now because de Sitter spacetime and Euclidean hyperbolic space have the same isometry group, the relevant representation theory has been extensively developed [77]. The scalar representations fall into three families, the principal, complementary, and discrete series. The principal representations are those which, under contraction of the de Sitter group to the Poincaré group, correspond to the familiar flat space representations [73]. They are of two kinds, which in fact are classified precisely by the weights given above. The first kind is the case of complex weights: these of course violate the Strominger bound. The only other kind is precisely the case where the bound is saturated.

The complementary series consists of representations where the Strominger bound is satisfied but not saturated[14], but these representations have no flat-spacetime analogue. [The discrete series corresponds to the special, massless case [74][75].] We conclude that $\epsilon = 3$ is the *only* value which is both physically acceptable and at the same time corresponds to a well-defined flat space representation. It is interesting that $\epsilon = 3$ is singled out in this way, for this is physically perhaps the most important case: as we saw, it corresponds to zero-pressure, non-relativistic matter superimposed on an initial OVV spacetime.

Next we turn to our principal concern, the question of stability from the point of view of the Seiberg-Witten pair production process.

The Euclidean versions of these spacetimes are obtained by complexifying K as usual, so we have

$$g_s(\epsilon, \text{K}, \text{L})_{++++} = \text{dt}^2 + \text{K}^2 \sinh^{(4/\epsilon)}(\frac{\epsilon\,\text{t}}{2\text{L}})\,[\text{d}\theta_1^2 + \text{d}\theta_2^2 + \text{d}\theta_3^2]. \qquad (45)$$

This metric has, of course, the same asymptotic structure as the Euclidean OVV metric $g_{\text{OVV}}(2^{-(2/\epsilon)}\text{K}, \text{L})_{++++}$; it is an example of an asymptotically hyperbolic Riemannian metric. We can think of it as a deformation of the OVV metric. This is of course alarming, since, as was discussed in the Introduction, there are known examples of such deformations [equation (1)] which are strongly unstable against pair production of BPS branes [12]. We can investigate this issue by examining the action given in equation (27): we have

$$\text{S}(g_s(\epsilon, \text{K}, \text{L})_{++++}\,;\,\text{t}) = 8\pi^3\,\text{TK}^3[\sinh^{(6/\epsilon)}(\frac{\epsilon\,\text{t}}{2\text{L}}) - \frac{3}{\text{L}}\int_0^{\text{t}}\sinh^{(6/\epsilon)}(\frac{\epsilon\,\tau}{2\text{L}})\,\text{d}\tau]. \qquad (46)$$

In the case of non-relativistic matter [$\epsilon = 3$] this can be evaluated, the result being

$$\text{S}(g_s(3, \text{K}, \text{L})_{++++}\,;\,\text{t}) = 4\pi^3\,\text{TK}^3[\frac{3\text{t}}{\text{L}} + e^{(3\text{t}/\text{L})} - 1]. \qquad (47)$$

It is easy to see that this is *never negative*, and so there is no danger of Seiberg-Witten [13] instability here. In fact, this function increases away from its initial value, zero. Thus the deformation is innocuous here, as it was *not* in the case of the metric $g_c(2, 2\text{K}, \text{L})_{++++}$ [equation (1)].

In the general case, a simple calculation shows that the derivative of the brane action is given by

$$\frac{\text{d}}{\text{dt}}\text{S}(g_s(\epsilon, \text{K}, \text{L})_{++++}\,;\,\text{t}) = \frac{24\pi^3\,\text{TK}^3}{\text{L}}\sinh^{(6/\epsilon)}(\frac{\epsilon\,\text{t}}{2\text{L}})[\coth(\frac{\epsilon\,\text{t}}{2\text{L}}) - 1]. \qquad (48)$$

[14]The relevant Hermitian form in the complementary case [[77], page 518] involves a gamma function which is ill-defined if the Strominger bound is saturated, so $\epsilon = 3$ certainly does not belong to this series.

This is obviously positive. Since $S(g_s(\epsilon, K, L)_{++++}; 0)$ is zero, we see that the action will never be negative, and so there is no Seiberg-Witten instability here for any value of ϵ. All forms of matter that satisfy the Null Energy Condition help to stabilize the spacetime.

Recall that the brane action was constant in the case of pure OVV spacetime. What we have shown is that the introduction of matter satisfying the NRC actually makes the OVV space "more stable", in the sense that it converts this constant to an increasing function. By contrast, one can show [12] that NRC-violating geometry causes it to *decrease*. However, that is not conclusive, since a decreasing function can of course still be everywhere positive. This too was investigated in [12], and the conclusion is that a NRC-violating geometry *can* be stable, but only for certain values of the parameters. We shall discuss this in more detail in the next section.

To summarize, then, we have arrived at a somewhat paradoxical conclusion. The OVV spacetime [pictured as the triangle OAB in Figure 4] is well-behaved, but it is on the brink of two very different catastrophes. First, it is "nearly" singular in the sense that it is null geodesically incomplete, but only in a rather harmless way. Second, it is "nearly" unstable in the sense that a slight perturbation could set off the Seiberg-Witten "stringy instability". We have found that making the OVV spacetime more realistic — by introducing conventional matter and radiation into it — definitely staves off instability, yet at the same time causes the spacetime to become genuinely singular. Furthermore, this singularity is of a particularly persistent kind: no amount of SEC violation by a scalar with a conventional kinetic term, or distortion of the geometry, can remove it.

6. Avoiding the Singularity

We have stressed that the singularity in the metric $g_s(\epsilon, K, L)_{+---}$ [equation (35)] is not a mere consequence of the assumed symmetries; nor however can it be understood in terms of the classical singularity theorems based on the Strong Energy Condition. In fact, the only way[15] to remove this singularity is to violate the Null Ricci Condition [50][51], or the Null Energy Condition if we assume the validity of the Einstein equations.

A fundamental matter field which genuinely violates the NEC is very hard to handle, as has been discussed for example in [52][53][54][12][55]. It is true that, to avoid a singularity, we need only a brief period of NEC violation, but it would clearly be better to avoid the various unpleasant physical properties of NEC-violating matter.

It is here that the distinction between the NRC and the NEC is useful. The former is of course a purely geometric condition [see (26) above], while the latter refers only to items such as pressure and energy densities. The two are linked by the gravitational field equation. They are equivalent if the Einstein equations hold exactly, but of course this is a highly questionable assumption in the early Universe. In braneworld models [63][64][65] there are explicit corrections to the Einstein equation which allow the NRC to be violated *while every matter field satisfies the NEC*, so that one says that the NEC is "effectively" violated [67][12]. Similarly, the NEC and the NRC can be usefully different in certain Gauss-Bonnet theories [66].

If we arrange to violate the NEC only effectively, then we can hope to circumvent the Andersson-Galloway theorem without using dangerously unstable forms of matter.

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[15]See the Appendix.

However, there is a problem here: the Seiberg-Witten instability is determined purely by geometric data [rates of growth of volume and area] so it can still be present *even if the NEC is only violated effectively*. This fact is the most serious obstacle to removing the singularity at t = 0 in $g_s(\epsilon, K, L)_{+---}$, and the same comment holds for any spacetime with the OVV topology.

The question, then, is whether we can find an OVV-like spacetime in which all matter fields preserve the NEC, in which the metric violates the NRC, but which does not suffer from uncontrolled Seiberg-Witten pair-production. Ideally one should do this by deriving such a metric from a specific theory, such as in the brane-world models mentioned above [63][64][65]; but for clarity we shall just work with a simple family of metrics that violate the NRC, and try to determine whether there are parameter values allowing us to avoid any instability. A family of such metrics, defined on a space with OVV topology $\mathbb{R} \times T^3$, was discussed in [12], and will now be described.

Consider the metrics given by

$$g_c(\gamma, K, L)_{+---} = dt^2 - K^2 \cosh^{(4/\gamma)}(\frac{\gamma\, t}{2L})\,[d\theta_1^2 + d\theta_2^2 + d\theta_3^2], \qquad (49)$$

where γ is a positive constant and the c subscript refers to the cosh function. Here K is simply the minimal radius of the spatial torus. The similarity to $g_s(\epsilon, K, L)_{+---}$ is obvious; in particular, this spacetime is "asymptotically OVV". The Penrose diagram is again rectangular, as in Figure 5, with height determined by L and width determined by K; the only difference is that the bottom of the diagram is spacelike *but not singular*. Since these spacetimes satisfy all of the other conditions of the Andersson-Galloway theorem and have a future conformal infinity with toral topology, we see that they must violate the Null Ricci Condition. In fact, these metrics were introduced in [56] precisely in order to demonstrate explicitly that "phantom" cosmologies [62] need not be singular. They are obtained in the same way as $g_s(\epsilon, K, L)_{+---}$, that is, by introducing "matter" with a *constant* equation-of-state parameter into a de Sitter-like background; the difference is that this "matter" has negative energy density. These spacetimes *always* [for all γ and at all times] violate the NRC. Thus they are a kind of NRC-violating, non-singular "dual" to the $g_s(\epsilon, K, L)_{+---}$ spacetimes.

Consider the Seiberg-Witten brane action $S(g_c(\gamma, K, L)_{++++}; t)$ for the Euclidean version of this metric, $g_c(\gamma, K, L)_{++++}$, obtained as usual by complexifying K. Clearly, since the area of a cross-section is never zero, $S(g_c(\gamma, K, L)_{++++}; 0)$ is positive; with tension T, the value is $8\pi^3 T K^3$. However, again oppositely to the case in which the NRC is satisfied, the action immediately decreases, and in fact it always decreases for all values of t > 0. The question as to whether it remains positive as $t \to \infty$ is therefore settled by computing the limiting value of the action. Unlike the action itself, this limit can be evaluated explicitly. It turns out [12] that it is actually negatively divergent for all values of $\gamma \leq 3$. [This is why the metric discussed in the introduction, $g_c(2, K, L)_{++++}$, suffers from Seiberg-Witten instability.] For values of $\gamma > 3$ one can show that

$$\lim_{t \to \infty} S(g_c(\gamma, K, L)_{++++}; t) = 2^{(3 - \frac{6}{\gamma})} \pi^3 K^3 T \frac{(\gamma - 6)(3 + \gamma)}{\gamma(\gamma - 3)}. \qquad (50)$$

This is indeed negative for values of γ strictly between 3 and 6. The spacetime will be unstable for these values, even if the NEC violation is merely effective. But for values of

γ greater than or equal to 6, the limit is either positive or zero, and therefore the action is strictly positive. Thus for example the spacetime with OVV topology and metric

$$g_c(6,\ \text{K},\ \text{L})_{+---}\ =\ dt^2\ -\ K^2\cosh^{(2/3)}(\frac{3\,t}{L})\,[d\theta_1^2\ +\ d\theta_2^2\ +\ d\theta_3^2] \tag{51}$$

violates the NRC and yet is entirely stable against brane pair-production: in fact the brane action in this case is simply

$$S(g_c(6,\ \text{K},\ \text{L})_{++++}\,;\ t)\ =\ 8\pi^3 K^3 T\, e^{(-3t/L)}. \tag{52}$$

Thus we see that we can indeed violate the NRC without destabilizing the spacetime, and that this does remove the singularity.

Since we wish to use this construction to remove the singularity, it is natural to assume that it is relevant only to the very earliest, pre-inflationary stages. Therefore we wish to choose the parameters such that the spacetime naturally evolves to an inflationary metric with the correct inflationary curvature radius, L_{INF}. The OVV metric itself naturally has this [local] inflationary structure if we choose $L = L_{INF}$ — that is, the OVV metric is [locally] just the one that is usually called the inflationary metric. Furthermore, we know that the metrics we have been considering do evolve to the OVV metric, so we automatically have all the ingredients we need. We must however determine how to choose the parameter γ so that the NRC-violating phase is brief.

If we take the unit timelike vector associated with proper time in equation (49) then we can, with the help of an arbitrary unit spacelike vector perpendicular to it, construct a null vector k^μ; then it can be shown [12] that the corresponding Ricci tensor satisfies

$$R_{\mu\nu}\, k^\mu\, k^\nu\ =\ \frac{-\gamma}{L^2}\operatorname{sech}^2(\frac{\gamma t}{2L}). \tag{53}$$

The function on the right side therefore measures the extent of NRC violation. For fixed L and t, this expression tends to zero as γ becomes large, yet its integral from 0 to ∞ is $-2/L$, independent of γ; thus the effect of taking γ to be large is to focus the NRC violating effect close to $t = 0$. Since "large" values of γ — values greater than or equal to 6 — are precisely what we need for stability, we can summarize this discussion as follows: NRC violation does not destabilize the spacetime *provided that it takes place over a period of time which is sufficiently brief.* Since the magnitude of $R_{\mu\nu}\, k^\mu\, k^\nu$ falls to half of its maximal value in a time

$$t_{1/2}\ =\ \frac{2\,L}{\gamma}\cosh^{-1}(\sqrt{2}), \tag{54}$$

we see that, to be specific, instability will be averted if we can arrange for the NRC-violating effect to decay so rapidly that $R_{\mu\nu}\, k^\mu\, k^\nu$ falls to half its maximal magnitude in a time no longer than $(L/3)\cosh^{-1}(\sqrt{2})$. As we have stressed, a brief period of NRC violation is exactly what we want in an inflationary picture, where $L = L_{INF}$; in fact, we might prefer it to be even shorter than $(L_{INF}/3)\cosh^{-1}(\sqrt{2})$, that is, we might like γ to be even larger than 6. However, the value 6 is singled out as the smallest one which is physically acceptable. The precise value of γ has to be fixed by quantum gravitational considerations.

Let us assemble the pieces of the puzzle. The basic idea is that, in some model [such as a braneworld model] in which the Einstein equations are corrected at very early times, the NEC and the NRC initially fail to coincide : all of the matter fields obey the NEC at all times, but the NRC is violated at these early times. For a spacetime with the OVV topology, the result is a non-singular metric which resembles $g_c(\gamma, K, L_{INF})_{+---}$. There will be no instabilities if γ is at least 6; in fact, such values of γ also ensure that the NRC-violating era is quickly replaced by an inflationary period.

Very soon, then, $g_c(\gamma, K, L_{INF})_{+---}$ becomes indistinguishable from the OVV metric $g_{OVV}(2^{-(2/\gamma)}K, L_{INF})_{+---}$ [equation (22)], which is, locally, the standard *inflationary* metric. Notice that one relic of the NRC-violating era survives: the parameter K, which measures the radius of the torus at its smallest. Thus K is some extremely small number.

This inflationary era ends in the conventional way at time t_{EXIT}, and we switch [via some transitional geometry which we shall not attempt to describe] to a metric like $g_s(4, K_{EXIT}, L_{DE})_{+---}$ [equation (35)] to describe the radiation-dominated era. Here K_{EXIT} is related to the size of the torus at time t_{EXIT}, and L_{DE} is the length scale appropriate to the current Dark Energy phase — that is, L_{DE} is very much larger than L_{INF}. At some time, before decoupling, $g_s(4, K_{EXIT}, L_{DE})_{+---}$ in its turn will be replaced by a metric like $g_s(3, K_{RAD}, L_{DE})_{+---}$, where K_{RAD} is related to the size of the torus at the end of the radiation-dominated era. This is the metric appropriate to the matter-dominated era. Finally, $g_s(3, K_{RAD}, L_{DE})_{+---}$ will eventually become indistinguishable from yet another OVV metric, as the matter dilutes and the Universe enters its final phase, in which the dark energy dominates.

This picture is particularly natural if we are using the so-called "low-scale" versions of inflation, in which the inflationary era begins at a mass scale about 3 orders of magnitude below the Planck mass. These arise naturally in string theory, but there are difficulties in understanding the initial conditions for this kind of inflation. However, Linde [4][5] stresses that a simple and elegant way to solve the problem of initial conditions for low-scale inflation is to assume that the spatial sections of the Universe are compact and have either zero or negative curvature. As discussed in [9], the negative case is not acceptable in string theory because such spacetimes are unstable in the Seiberg-Witten sense. Thus we are left with the flat case, as discussed in this work. The flat, compact spatial sections solve the problem provided that the system is able to sample an entire spatial section. This will happen if the relevant part of the Penrose diagram of the early Universe is much taller than it is wide. Since any inflation-like expansion naturally expends large amounts of conformal time, and since the parameter K is very small, it should not be very difficult to arrange this. Thus the whole picture is ideally suited to incorporate Linde's ideas, though of course it must be made numerically much more precise; in particular, one needs a quantum-gravitational account of the shape of the Penrose diagram describing the earliest phases of cosmic evolution.

A Penrose diagram illustrating the essential points of this discussion is given in Figure 6. The horizontal line in Figure 6 represents decoupling; the lower dot represents a particle or gravitational perturbation in the early Universe which has sampled an entire spatial section, as in Linde's scenario; and the upper dot represents an observer like ourselves, who is not *yet* able to see an entire torus, though he will be able to do so in future. There is much more to be said about this picture, but let us turn to a more pressing issue: what
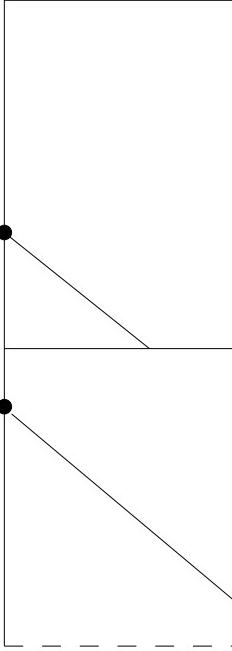
Figure 6: Penrose diagram of the Toral Universe

happens before t = 0, represented by the dashed line at the bottom?

Here, again, we can seek guidance from Ooguri et al [1]. In their discussion, the answer has two aspects, one Euclidean, the other a mixture of Euclidean and Lorentzian.

In the Euclidean case, Ooguri et al note that the region of large negative ζ in their metric given in (19) involves circles of arbitrarily small radius. Moving towards still smaller circles should be interpreted as motion towards *larger* circles in some T-dual theory. For example, Ooguri et al begin with a IIB theory, so the dual description should be in terms of IIA theory. In cosmological language, this should mean that all physics on tori "earlier" than t = 0 is completely equivalent to dual physics on later tori: the past of t = 0 is equivalent to its future, described in terms of different variables. In short, the full Euclidean manifold describing our Universe in the OVV picture consists of two identical "trumpet-shaped" spaces joined together smoothly at their narrowest point. The metric near this point should resemble $g_c(\gamma, \text{K}, \text{L})_{++++}$, for some large value of γ.

This is in fact very similar to the standard way of describing the manner in which *string gas cosmologies* [7] avoid being singular. Clearly it would be highly desirable to give a precise description of the mapping of variables under T-duality in cosmology, as Ooguri et al do for their manifold [in terms of D3 branes on the IIB side and D2 branes on the IIA side]. Cosmology could actually implement string duality not just as a mathematical equivalence, but in terms of spacetime geometry.

Ooguri et al are also interested in a possible interpretation in terms of the Hartle-Hawking construction [2]. This would involve regarding t = 0 as the section where there is a transition from Euclidean geometry to Lorentzian. Recall that we argued in Section 2 that *both* ways of complexifying a Euclidean space should have complementary roles in string cosmology. If we take equation (49) and complexify *time* instead of the spatial

parameter K, we obtain a metric of completely negative signature:

$$g_c(\gamma, \text{K}, \text{L})_{----} = -\,d\tau^2 - \text{K}^2 \cos^{(4/\gamma)}(\frac{\gamma\,\tau}{2\text{L}})\,[d\theta_1^2 + d\theta_2^2 + d\theta_3^2]. \qquad (55)$$

Of course, such a metric is just an unusual way of describing a Euclidean space; one has to take the absolute value before computing a length, just as one does for spacelike displacements in Minkowski space with signature $(+ - - -)$. [Note however that a sphere has negative curvature in this signature.] Here τ is a coordinate which takes its values in the interval $(-\pi\text{L}/\gamma, 0]$. The space is foliated by tori, which reach a maximal radius of K at $\tau = 0$. Hence, this Euclidean space can be smoothly joined [at $t = \tau = 0$] to the Lorentzian space described by $g_c(\gamma, \text{K}, \text{L})_{+---}$ [equation(49)]. It is significant that complexifying t in $g_c(\gamma, \text{K}, \text{L})_{+---}$ does not complexify the metric, as would have happened if we had complexified time in $g_s(\epsilon, \text{K}, \text{L})_{+---}$ [equation (35)] for $\epsilon = 3$ or 4. That is, the early period of NRC violation reduces the extrinsic curvature of the spatial slices to zero as the transition is approached from the Lorentzian side, thus naturally allowing a smooth transition to a Euclidean regime while removing the imaginary factor from the metric when time is complexified. The two different forms of analytical continuation used here are indeed complementary: one complexifies K in $g_s(\epsilon, \text{K}, \text{L})_{+---}$ to understand the physics of the boundary, and one complexifies t in $g_c(\gamma, \text{K}, \text{L})_{+---}$ to understand the earliest phases of this universe.

Thus, another possible answer to the question as to what happens at $t = 0$ is that time ceases to have any meaning if we try to probe beyond that point: the spacetime is replaced by the Euclidean space with metric $g_c(\gamma, \text{K}, \text{L})_{----}$. It would be very interesting to attempt to construct the corresponding wave function, in the manner described in [1]. Note that $g_c(\gamma, \text{K}, \text{L})_{----}$ is singular at $\tau = -\pi\text{L}/\gamma$, but it is not entirely clear what this means. Note that, in the absence of a meaningful "time" in the Euclidean region, there is no sense in which the Universe can be said to "evolve" from this point, so it may not be as obnoxious as the more familiar kinds of cosmological singularity. Nor does the singularity persist into the Lorentzian regime, as happens in the case of the well-known Hawking-Turok instanton [78]. Nevertheless one would like to know whether there is some way of smoothing over this Euclidean singularity, perhaps after the manner of [79]. This point remains to be investigated in detail.

7. Conclusion

The main points of our discussion can be summarized as follows. Ooguri, Vafa, and Verlinde have proposed a very unusual multiple interpretation of the locally hyperbolic space $\text{H}^2/\mathbb{Z}$, and have explored the corresponding physics with a view to interpreting a Lorentzian version as an accelerating cosmology. We have extended this to the four-dimensional case, $\text{H}^4/\mathbb{Z}^3$. After clarifying the mathematics, we found that the cosmology determined by this space is null geodesically incomplete. This foreshadows the fact that any more realistic version of these spacetimes is bound to be singular unless the Null Ricci Condition is violated in the earliest Universe. [It is worth noting that, however one proceeds, the presence of *even one circle* in the spacelike sections will lead, via the Andersson-Galloway theorem, to the conclusion that the NRC has to be violated if singularities are to be avoided. That is, almost any procedure of the OVV kind is likely

to lead to this conclusion.] We argued that the NRC can however be violated without inducing instability, and we proposed concrete ways in which the initial singularity can be avoided.

Ooguri et al establish a kind of duality between an accelerating cosmology, represented by $H^2/\mathbb{Z}$, and a particular black hole configuration. This is done by embedding $H^2/\mathbb{Z}$ in string theory, through a compactification of the form $(H^2/\mathbb{Z}) \times S^2 \times CY$. The obvious analogue in the case of $H^4/\mathbb{Z}^3$, which has the local geometry of a space of constant negative curvature, is a space of the form $(H^4/\mathbb{Z}^3) \times FR$, where FR denotes a Freund-Rubin space. Freund-Rubin compactifications have recently been revived [25][26][27] and it would be interesting to study OVV cosmology from that point of view. In particular, one should study the "OVV dual" of the Freund-Rubin embedding of our cosmologies. Note that Freund-Rubin compactifications themselves have a dual interpretation in terms of three-dimensional conformal field theories [80].

One striking feature of our analysis has been the remarkable way in which the introduction of matter affects the OVV spacetime, rendering it non-perturbatively stable in string theory when the NRC is satisfied [and even — sometimes — when it is not]. Now Firouzjahi et al [81] have argued that the introduction of matter into de Sitter space forces a modification of the Hartle-Hawking wave function [as also do higher metric modes]. Clearly it would be desirable to reconsider the findings of Ooguri et al from this point of view. The spacetimes we have considered here provide a concrete background for such an investigation.

We have stressed that the OVV construction introduces a new length scale when H^2 or H^4 is partially compactified[16]; this is the parameter K in $g_s(\epsilon, K, L)_{+---}$ [which determines the K in $g_c(\gamma, K, L)_{+---}$ when the latter is used in the way we have suggested]. The ratio L/K determines the shape of the Penrose diagrams for these spacetimes, and this shape is of profound physical significance [72]. One would certainly hope that it is fixed by the wave function of the Universe, and proving that the wave function is peaked at a physically reasonable value of L/K is an important challenge for these theories.

Acknowledgements

Appendix: About The Andersson-Galloway Theorem

In this appendix we briefly explain the terminology used by Andersson and Galloway [50][51], and comment on the conditions assumed in their theorem used above. This is important, because our argument is based on the claim that there is only one way to circumvent the theorem — to violate the Null Ricci Condition.

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[16]Strictly speaking, there are actually three such parameters in the H^4 case.

A four-dimensional spacetime M_4 with Lorentzian metric g_M is said to have a *regular future spacelike conformal boundary* if M_4 can be regarded as the interior of a spacetime-with-boundary X_4, with a [non-degenerate] metric g_X such that the boundary is *spacelike* and lies to the future of all points in M_4, while g_X is conformal to g_M, that is, $g_X = \Omega^2 g_M$, where $\Omega = 0$ along the boundary but $d\Omega \neq 0$ there. This is just a technical formulation of the idea that the Penrose completion should be well-behaved. There are examples of accelerating spacetimes which do not have a regular future spacelike conformal boundary — the Nariai spacetime [82] is of this type — but these spacetimes typically violate the NEC [*some* "phantom" cosmologies, though certainly not all [56], are singular towards the future, with the singularity at finite proper time] or are highly non-generic, like Nariai spacetime [which is on the very brink of having a naked singularity]. One certainly should not hope to escape from the conclusions of the Andersson-Galloway theorem by resorting to such examples.

A spacetime is said to be *globally hyperbolic* if it possesses a Cauchy surface, that is, a surface on which data can be prescribed which determine all physical fields at later and earlier events in spacetime, since all inextensible timelike and null curves intersect it. This forbids naked singularities, but it also disallows ordinary AdS. The dependence of the Andersson-Galloway theorem on this assumption might lead one to ask whether our conclusions can be circumvented by dropping global hyperbolicity. This is a topical suggestion, since it has recently been claimed [83] that string theory allows spacetimes which violate global hyperbolicity even more drastically than AdS. To see why this too will not work here, we need the following definition.

A spacetime with a regular future spacelike conformal completion is said to be *future asymptotically simple* if every future inextensible null geodesic has an endpoint on future conformal infinity. This just means that there are no singularities to the future — obviously a reasonable condition to impose in our case, since it would be bizarre to suppose that singularities to the future can somehow allow us to avoid a Big Bang singularity. Andersson and Galloway [50] show, however, that if a spacetime has a regular future spacelike conformal completion and is future asymptotically simple, *then it has to be globally hyperbolic.* Thus it would not be reasonable to drop this condition in our context. Notice that this discussion has a more general application: it means that, if the future of our Universe resembles that of de Sitter spacetime, any attempt to violate global hyperbolicity will necessarily cause a singularity to develop to the future. It would be interesting to understand this in the context of [83].

Finally, one might wonder whether a compact flat three-manifold can in fact have a first homology group which is pure torsion. The rather surprising answer is that it can: there is a unique manifold of this kind, the *didicosm*, described in [84][9]. The didicosm has the form $T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$, that is, it is a quotient of the three-torus. However, if it were possible to construct a singularity-free spacetime metric on $\mathbb{R} \times T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$ without violating the NRC, this metric would pull back, via an obvious extension of the covering map $T^3 \to T^3/[\mathbb{Z}_2 \times \mathbb{Z}_2]$, to a non-singular metric on $\mathbb{R} \times T^3$, also satisfying the NRC. This is a contradiction. Similarly, of course, one cannot escape the conclusions of the theorem by allowing spacelike future infinity to be non-orientable.

We conclude that the only physically reasonable way to avoid a Bang singularity in a spacetime of OVV topology is indeed to violate the NRC.

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